Icarus 412 (2024) 115970

Contents lists available at ScienceDirect

Icarus

journal homepage: www.elsevier.com/locate/icarus

Assessment and review of modern lunar crescent visibility criterion

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ARTICLE INFO

Keywords:
Assessment
Lunar crescent
Visibility criterion
Database: Contradiction rate
Descriptive statistics
Regression analysis

ABSTRACT

The lunar crescent visibility criterion study is one of the most nontrivial discussions in the field of astronomy. A lunar crescent visibility criterion is used for calendrical determination, and a different lunar crescent visibility criterion results in different dates of calendar. This paper endeavors to provide an assessment for modern lunar crescent visibility criterion using swarm plot analysis, contradiction rate analysis, and regression analysis, based on 8290 collected data of lunar crescent sighting. The result analysis provides new comparative insight on modern lunar crescent visibility criterion, and suggest a new criterion based on the data of lunar crescent sighting.

1. Introduction

Lunar crescent visibility criterion is a criterion where a lunar crescent is assumed to be sighted. A lunar crescent visibility criterion is constructed based on data of lunar crescent sighting. The larger the coverage of the data; location, and variation wise, the more reliable a lunar crescent visibility criterion becomes (Ahmad et al., 2022). Furthermore, criterion of a lunar crescent visibility is based on assumption, and assumption is heterogenous from group to another (Faid et al., 2023). Indonesia Muhammadiyah adopts *Wujudul Hilal* as their basis for lunar crescent visibility criterion, where any lunar crescent is assumed to be sighted as long as it is located above the horizon. Istanbul's lunar crescent visibility criterion adopts upper range of visibility lunar crescent, where a lunar crescent has extremely high chance of sighting when it is located above Istanbul lunar crescent visibility criterion. MABIMS (Brunei, Indonesia, Malaysia, and Singapore Informal Meeting of Religious Minister) lunar crescent visibility criterion both in 1995 and 2021, in the meantime is based on lower line of lunar crescent sighting, where a lunar crescent has lower chance of visibility even it is located above the criterion (Maskufa et al., 2022). Apart from Muhammadiyah, Istanbul and MABIMS lunar crescent visibility criterion, there are numbers of lunar crescent visibility criterion that exist (Mufid and Djamaluddin, 2023). To date, there are at least 15 lunar crescent visibility criterion that have been found in the literature since 1901. Table 1 portrays a collection of modern lunar crescent visibility criterion since 1910 Fotheringham criterion.

There are numerous lunar crescent visibility criterion, not counting those who published outside the medium of journal and academic literature, each of the criterion is unique from one to another without any means to measure and compare each of the criterion performance in predicting lunar crescent visibility and for calendrical determination. There are a number of studies that conducted assessment of the lunar crescent visibility criterion. The first to assess the lunar crescent visibility criterion is Ilyas. Ilyas' works on lunar crescent visibility criterion was concluded with his conclusive study, which was published in 1994, entitled *Lunar Crescent Visibility and Islamic Calendar*. In this assessment, Ilyas used the literature analysis method to analyze the works of various lunar crescent visibility criteria, by comparing them against his published literature on lunar crescent visibility. Ilyas' qualitative assessment of lunar crescent visibility appeared to favor his own interpretation on lunar crescent visibility criterion, as the literature assessment is heavily based on his own literature works and not strongly supported by any lunar crescent sighting data (Ilyas, 1994). Another assessment of lunar crescent visibility criterion, this time by Schaefer, was conducted

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https://doi.org/10.1016/j.icarus.2024.115970
Received 21 November 2023; Received in revised form 18 January 2024; Accepted 19 January 2024
Available online 21 January 2024
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shortly after. Schaefer in 1996 published an assessment of lunar crescent visibility criterion entitled "Lunar Crescent Visibility", and alongside with another paper authored with Leroy Doggett in 1994 with the same title. Schaefer's assessment was based on five Moonwatch projects and other nationwide lunar crescent sighting projects, involving 2000 participants and 294 data of lunar crescent sightings across North American longitudes. The substantial number of data enabled Schaefer to assess various factors that contributed to lunar crescent visibility, including atmospheric factors, optical factors, and human factor. In assessing lunar crescent visibility criterion, Schafer adopted the method of histogram bias analysis where he assessed the reliability of moon age, lag time, and differences in altitude. However, Schaefer's assessment was only limited to these three parameters, while there are a number of other lunar crescent visibility criteria available during his time which were not assessed (Schaefer, 1996; Schaefer et al., 1992).

The assessments of lunar crescent visibility criteria continued in 1998 in the form of a PhD thesis. Louay Fatoohi published a PhD thesis where he assessed the lunar crescent visibility criteria based on a collection of 506 lunar crescent sighting reports from ancient, medieval, and modern times. Fatoohi used a combination of lunar crescent sighting records analysis, literature analysis, and contradiction rate analysis to assess lunar crescent visibility criteria. This makes his assessment the most comprehensive assessment even to this day. Fatoohi assessed fifteen lunar crescent visibility criteria, including ancient ones which are Babylon and Hindus criteria, medieval lunar crescent visibility criteria which are Khawarizmi, al-Qallas, al-Lathiqi, al-Sanjufini, ibn Yunus, and Maimonides criteria, and modern lunar crescent visibility criteria which are Danjon, Fotheringham, Maunder, Bruin, Ilyas, and Yallop criteria. The only minor downside to Fatoohi's lunar crescent visibility criterion is time, as his lunar crescent sighting data collection and criterion assessment are only limited to that which are available during his time. In the present day, over 5000 data of lunar crescent sightings can be collected and there are numerous lunar crescent sighting criteria that developed after 1998. Fatoohi's comprehensive method to assess the lunar crescent visibility criterion should be re-emulated with more data of lunar crescent sightings and modern lunar crescent visibility criteria (Fatoohi, 1998).

Another assessment is lunar crescent analysis. Lunar cycle analysis involves examining the criteria for the visibility of the lunar crescent, focusing on the naturalness of these criteria in relation to the monthly phases of the moon. The analysis includes comparison on the lunar crescent visibility criteria on full moon cycle, and comparison of the criteria based on 29 or 30 Hijri month cycle. Example of lunar cycle analysis is Rodzali and Man (2021) where they asses the frequency of 29th and 30th in over 20 years of Hijri calendar. Another example of lunar cycle analysis is Rahimi and Zainal (2019) where they asses the MABIMS lunar crescent visibility criteria based on its accuracy of full month during 15th of a Hijri month, sunset and moonset. The issue with lunar cycle analysis is that the analysis is not justifiable. Lunar cycle is inconsistent from one lunar cycle to another. Each lunar cycle is unique to one to another depending on sun and moon declination, location of the observer involving elevation, light pollution and atmospheric

Table 1
Modern Lunar Crescent Criterion from Selected Publications.

No	Parameter	Source	Lunar Crescent Criteria Expression
1	**Altitude & Azimuth**	(Fotheringham, 1910, 1921)	$ArcV \geq 12.0 - 0.008DAZ$
2		(Maunder, 1911)	$ArcV \geq 11 - 0.005DAZ - 0.01DAZ^2$
3		(Ilyas, 1988)	$ArcV = -0.0027356815\,DAZ - 0.0136648716\,DAZ^2 + 0.0002119205\,DAZ^3 + 10.2832719598$
4		(Fatoohi, 1998)	$ArcV_{Upper\ Limit} = 10.7638 + 0.0356\,DAZ - 0.0164DAZ^2 + 0.0004DAZ^3$ $ArcV_{Lower\ Limit} = 9.2714 - 0.0644\,DAZ - 0.0058DAZ^2 + 0.0002DAZ^3$
5		(Krauss, 2012)	$ArcV^{Athenian} = 0.0291254840\,DAZ + -0.0098347831\,DAZ^2 + 0.0000475196\,DAZ^3 + 10.5981838905$
6	**Arc of Light & Arc of Vision**	(Mohd Nawawi et al., 2015)	$ArcL \geq 3^\circ \& MAlt \geq 2^\circ$
7		(Rodzali, 2021)	$ArcL \geq 8^\circ \& MAlt \geq 5^\circ$
8		(Azhari, 2021)	$ArcL \geq 6.4^\circ \& MAlt \geq 3^\circ$
9	**Elongation**	(Danjon, 1936)	$ArcL \geq 7^\circ$
10		(Ilyas, 1984)	$ArcL \geq 10.5^\circ$
11		(McNally, 1983)	$ArcL \geq 5^\circ$
12		(Schaefer, 1991a, 1991b)	$ArcL \geq 7.5^\circ$
13		(Fatoohi et al., 1998)	$ArcL \geq 7.5^\circ \textit{for optical aided}$, $ArcL \geq 9.1^\circ \textit{for naked eye}$
14		(Odeh, 2004)	$ArcL \geq 6.4^\circ \textit{for optical aided}$, $ArcL \geq 7.7^\circ \textit{for naked eye}$
15		(Sultan, 2007)	$ArcL \geq 5^\circ$
16		(Hasanzadeh, 2012)	$ArcL \geq 5^\circ$
17	**Arc of Vision & Lunar Width**	(Bruin, 1977)	$ArcV = 11.5621745317 - 7.944238328\,W + 3.2608487770\,W^2 - 0.4559413249\,W^3$
18		(Yallop, 1998)	$q = \left(ArcV - 11.8371 + 6.3226W' - 0.7319W'^2 + 0.1018W'^3\right)/10$ $\textit{If } q > +0.216, \textit{Lunar Crescent Easily Visible} \textit{If } 0.216 \geq q > -0.014, \textit{Lunar Crescent Visibile Under Perfect Condition} -0.014 > q > -0.160, \textit{May Need Optical Aid to Find Crescent} -0.160 > q > -0.232, \textit{Will Need Optical Aid to Find Crescent} -0.232 > q > -0.293, \textit{Not visible with a telescope}, ArcL < 8.5^\circ -0.293 > q, \textit{Not visible}, \textit{below Danjon limit}, ArcL < 8^\circ$
19		(Odeh, 2004)	$V = ArcV - \left(-0.1018W^3 + 0.7319W^2 - 6.3226W + 7.1651\right)$ $V \geq 5.65$: Crescent is visible by naked eyes$2 \leq V < 5.65$: Crescent is visible by optical aid, and it could be seen by naked eyes. $-0.96 \leq V < 2$: Crescent is visible by optical aid only$V < -0.96$: Crescent is not visible even by optical aid.
20		(Qureshi, 2010)	$S = \left(ArcV - 0.351964\,W^3 + 2.222075\,W^2 - 5.422643\,W + 10.43418\right)/10$ $s > 0.15$: Easily Visible (EV)$0.05 < s < 0.15$: Visible under perfect conditions (VUPC) $-0.06 < s < 0.05$: May require optical aid to find crescent (MROA) $-0.16 < s < -0.06$: Require optical aid (ROA)$s < -0.16$: Not visible with optical aid (I)
21		(Alrefay et al., 2018)	$ArcV_{Naked\ Eye} > 9.34 - 4.51W + 3.3W^2 - 1.01W^3 ArcV_{Optical\ Aided} > 7.83 - 4.35W + 3.22W^2 - 1.02W^3$
22	**Lag Time**	(John, Calwell, 2011)	$lag\ (min) > -0.9709\ ArcL + 44.65\ \textit{for naked eye sighting}$ $lag\ (min) > -1.9230\ ArcL + 43.13\ \textit{for optical aided}$
23		(Gautschy, 2014)	$LT = 0.3342328913\,DAZ + -0.0715608980\,DAZ^2 + 0.0009924422\,DAZ^3 + 33.8890455442$

extinction, method of observation involving optical aided instrument setup, experience of observer and validity of a lunar crescent sighting, position of the earth in respect to the sun, and earth rotational speed. In addition, there is no practical implication in using lunar cycle to assess the lunar crescent visibility criteria. Assessment of lunar crescent visibility criteria must be based on the criteria performance in predicting lunar crescent sighting visibility and its practicality in determining the Hijri calendar. Lunar cycle analysis does not consider a criterion performance in predicting lunar crescent sighting and practicality for Hijri calendar.

Then, there is histogram bias analysis. Histogram bias analysis is an analysis of lunar crescent visibility criterion contradiction rate, in the form of histograms (Doggett et al., 1994). The histograms use y-axis as the contradiction rate, and x-axis as the lunar crescent visibility parameter. The histogram analysis enables the contradiction rate of lunar crescent sightings to be studied throughout the increment of the parameter. It highlights the point of the lunar crescent visibility criterion's weaknesses.In the analysis of histogram bias, it is deemed reliable for lunar crescent visibility criteria only if it exhibits a bias error of less than 50%. Criteria with substantial bias introduce heightened uncertainty parameters, rendering them impractical for calendrical purposes. Doggett et al. histogram bias analysis is, however, only applicable for zero order or singular parameter of lunar crescent visibility criterion. It is not applicable for multi-parameters of a lunar crescent visibility criterion. As modern lunar crescent visibility criterion is composed of multi-parameters, histogram bias analysis is not applicable for assessment of lunar crescent visibility criterion.

After the assessment work done by Ilyas, Schaefer, and Fatoohi on lunar crescent visibility criterion, there has been limited work dedicated to the assessment of lunar crescent visibility criterion. The current assessment, as discussed, is based on a limited locality, and does not portray actual observation of lunar crescent sighting. Therefore, there is a need to provide an assessment for lunar crescent visibility criterion. The assessment of lunar crescent visibility criteria is an assessment to measure how dependable the lunar crescent visibility criteria is in predicting the lunar crescent sighting. The assessment also can used to determine the practicality of a lunar crescent visibility criteria in determining the new Hijri month. Lunar crescent visibility criteria without any assessment cannot be analyzed comparatively as different lunar crescent visibility criteria use different parameters, expression, and data. Therefore, it is an utmost importance to assess the lunar crescent visibility criteria.

2. Methodology

The data of lunar crescent sighting is structured, based on the following syntax, which are

a. Visibility of lunar crescent sighting (V);
  i. with (V) for visible,
  ii. (I) for invisible.
b. Methodology of sighting (M);
  i. with (NE) for naked eye,
  ii. (OA) for optical aided.
c. Imaging technique (I);
  i. with (NU) for not used,
  ii. (CCD) for charge-couple device observation
  iii. (T) for Digital Single Light Reflect (DSLR) telescopic observation.
d. Geometric Parameter;
  i. moon age; the interval time between conjunction and the time of observation (*MA*),
  ii. lag time: the interval time between sunset and moonset or moonrise and sunrise (*LT*),
  iii. arc of vision: the angular difference in altitude between the Sun and the Moon (*ArcV*),
  iv. arc of light: the angular separation (elongation) between the Sun and the Moon (*ArcL*),
  v. different in azimuth: The angular difference in azimuth between the Sun and the Moon (*DAZ*),
  vi. moon altitude: The angular distance of the Moon above the horizon (*MAlt*)
  vii. width: the width of the lit area of the Moon measured along the Moon's diameter. in arc seconds (*W*).

The basis of the graph is portrayed in Fig. 1.

The ephemeris is calculated using the Skyfield python library. Skyfield documentation suggests it uses an updated Model of Earth Nutation IAU 2000 and updated planetary theory model. Most of the astrometry library and algorithms refer IAU 1980 Model of Earth Nutation and VSOP87 planetary theory such as Jean Meeus Astronomical Library, NASA Horizon, Accurate Times, and MoonC 6.0. These astrometry libraries are relatively faster than modern astrometry libraries, but the accuracy is limited to 1 arcsecond. This makes Skyfield be more accurate and precise. Its accuracy and precision in determining celestial object position make it a dependable astrometry library for ephemeris calculation, particularly for lunar crescent data. All of the calculation considers refraction below the horizon, using SkyField refraction api for pressure altitude adjustment, and Meeus formula for refraction, which used standard value of temperature and atmosphere (10C degree and 1010 millibar) (Meeus, 1991).

Assessment of lunar crescent visibility criterion require amassed lunar crescent sighting data from various location, geometrical position, and atmospheric conditions, this is to ensure that the assessment able to present and measure the performance of a lunar crescent sighting at plethora of sighting situations. In this paper, a total of 8290 data of lunar crescent sighting is collected from various literature (Faid et al., 2023). This manuscript provide an standardized data computation for assessment, therefore all of the data is calculated based on topocentric position using Skyfield ascl:1907.024 (Rhodes, 2020), with sunset as reference calculation for new moon observation, and sunrise for old moon observation. Data of lunar crescent visibility based on best time does not represent most of the design of lunar crescent visibility criterion, while data of lunar crescent visibility based on geocentric model does not represent an actual observation of lunar crescent sighting. Thus, the computation of data does not include these two variables.

There are three types of analysis that were performed in this assessment. First is swarm plot analysis. Swarm plot analysis is employed since it is able to measure the statistical distribution of lunar crescent visibility criterion towards predicting visibility and invisibility of lunar crescent sighting. Next is contradiction rate analysis. Contraction rate analysis are previously employed by Fatoohi (1998) to provide a comparative assessment of lunar crescent visibility criterion in predicting lunar crescent visibility. Contradiction rate is based on large

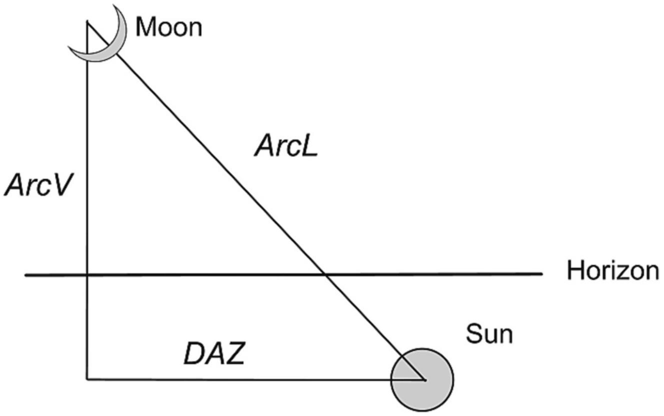

Fig. 1. Geometric Position of the Lunar Crescent During Observation.

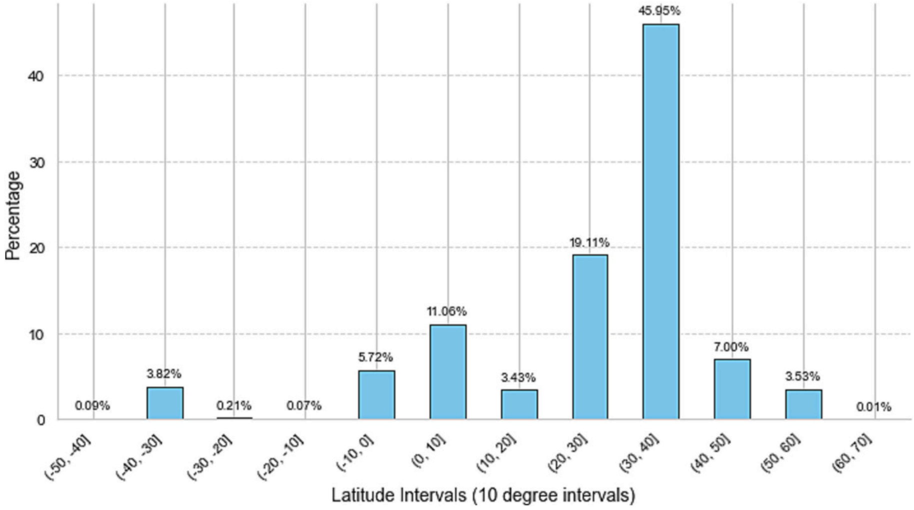

Fig. 2. Percentage of Lunar Crescent Sighting Data based on 10° Latitude Interval.

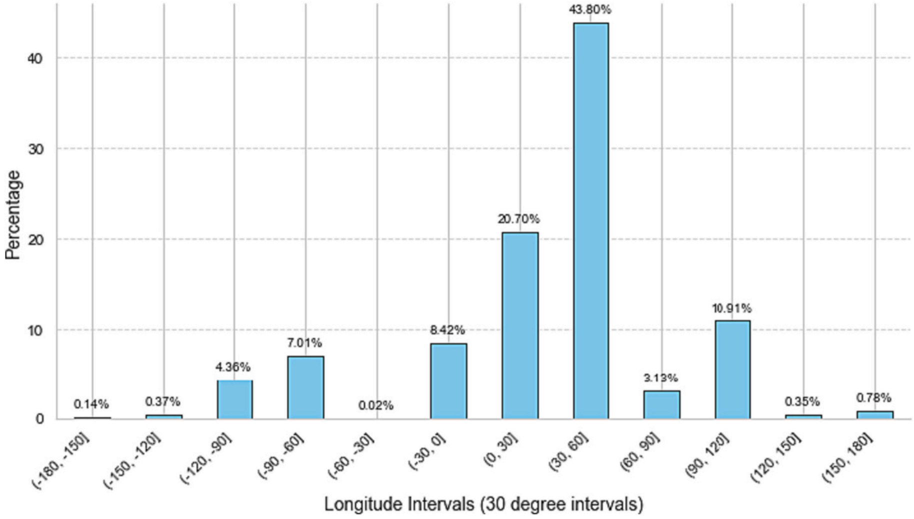

Fig. 3. Percentage of Lunar Crescent Sighting Data Based On 30° Longitude Interval.

amounts of data, which is suitable since this research has collected 8290 data of lunar crescent sighting. Regression analysis by Licht (1995) is employed to measure proximity of the lunar crescent visibility criterion lines towards minimum point of lunar crescent sighting. It can measure the ultimate accuracy of a lunar crescent visibility criterion as a dividing line between visibility and invisibility prediction. The assessment is conducted using HilalPy ascl:2307.031. For multiranges lunar crescent visibility criterion with the likes of Yallop dan Odeh, only criteria that provide that absolute visibility limit is included, which are "seen by naked eye" and "seen by telescope" are included in the ranking. Multiranges criterion that based on uncertainty such as 'May be seen by Naked Eye" and "May be seen by Telescope" is not included in the ranking. This is because it is not comparable to other lunar crescent visibility criteria designs and hinders the endeavor of forming an assessment of lunar crescent visibility criterion.

3. Result & discussion

3.1. Distribution of the collected data

The collected database contains 8290 records of lunar crescent visibility, 3023 records are negative records of lunar crescent sighting, while 5267 is positive record of lunar crescent sighting. Out of 5267 positive records of lunar crescent sighting, 4092 is naked eye lunar crescent sighting, and 1175 is optical aided lunar crescent sighting. Out of 1175 Optical aided lunar crescent observation, 335 is captured digitally using CCD, 191 is captured digitally using telescope and 648 data of lunar crescent is observed without any digital imaging. Out of 3023 negative records of lunar crescent sightings, 3013 records are naked eye observations, and ten records are optical aided observations.

Most of the data is located from latitude of 30 to 40° and longitude 0 to 60°, indicating that most of the data is originated from Asia, and Middle East. This is because most of the data of lunar crescent were reported from Iran, Iraq and Saudi Arabia, which account for almost three thousands data of lunar crescent sighting. United States log fourth most reports of lunar crescent sighting at eight thousands data. Data from Equator region mostly originated from Sri Lanka, Algeria, and Indonesia, with Indonesia logged more than seven thousand data of lunar crescent sighting. The distribution of the data is portrayed in Fig. 2, Fig. 3 and Fig. 4.

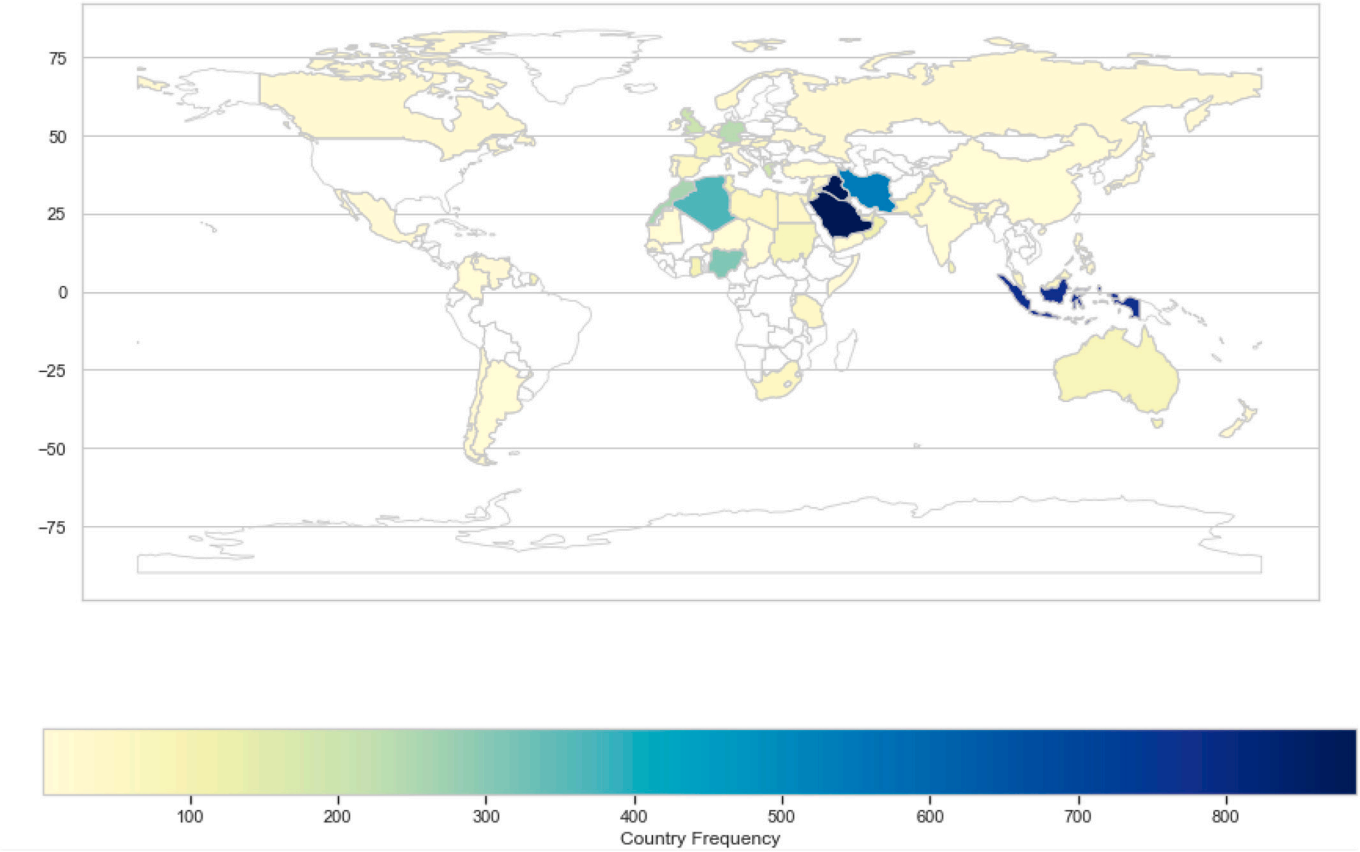

Fig. 4. Frequency of Reported Data of Lunar Crescent Sighting Based on Country.

3.2. Records of lunar crescent sighting

World records of lunar crescent sighting are as portrayed in Table 2. Only records with full report are accepted. Records of lunar crescent sighting without witness report or testimony are considered as fallible and therefore not accepted. Arc of vision is the altitude difference between sun and moon. The smallest arc of vision observed by observer is 0.46° by Martin Elsaesser. The new moon observation is conducted at 1100 m of elevation, using CCD imaging. The lag time of the moon is 2 min, with elongation of 15.21°, difference in azimuth of 15.21°, and 32.36 arc *sec* width. The smallest arc of vision observed by telescope is 1.29°, observed by Martin Elsaesser. The old moon observation has 21.85 degrees of elongation, 21.82 degrees of difference in azimuth, and 64.5 arc sec of width. The smallest arc of vision optically observed without any imaging technique is 3.01°, observed by Hossein Janghorbani from the Astronomy and Geophysics Center of Shahreza at 1902.05 m of elevation. The new moon observation has a lag time of 16.05 min, with elongation of 8.23°, difference in azimuth of 7.66°, and width size of 10.2 arc minutes. The position of the data is portrayed in Fig. 5.

The arc of light or elongation is the angle between sun and moon from the observer. The smallest arc of light observed by CCD is 4.67° by Martin Elsaesser. The new moon observation is conducted at 518.06 m of elevation, using CCD imaging. The lag time of the moon is 31.6 min, with arc of vision of 4.65°, difference in azimuth of 0.35°, and 2.93 arc *sec* width. The smallest arc of light observed by telescope is 5.98°, observed by Jim Stamm. The new moon observation has 4.85 degrees of arc of vision, 3.5 degrees of difference in azimuth, and 4.86 arc *sec* of width. The smallest arc of light optically observed without any imaging technique is 7.45°, observed by Alireza at 2080 m of elevation. The new moon observation has lag time of 38.23 min, with arc of vision of 6.99°, difference in azimuth of 2.58°, and width size of 8.4 arc minutes. The smallest naked eye observation for arc of light is 7.45° by John Pierce, with arc of vision of 7.44°, difference in azimuth of 0.52°, and width size of 8.22 arc *sec*, with lag time of 38.57 min. The position of the data is portrayed in Fig. 6.

Lag time is the time in minutes between sunset to moonset for new moon observation, or moonrise to sunrise for old moon observation. The smallest lag time observed by observer is 2 min by Martin Elsaesser. The new moon observation is conducted at 1100 m of elevation, using CCD imaging. The arc of light of the moon is only 0.46°, with elongation of 15.21-degrees, difference in azimuth of 15.21°, and 32.36 arc *sec* width. The smallest lag time observed by telescope is 13.7 min, observed by Abbas Ahmadian. The new moon observation has 2.63 degrees of arc of vision, 9.64 degrees of arc of light, 9.27 degrees of difference in azimuth, and 12.55 arc sec of width. The smallest lag time optically observed without any imaging technique is 16.05 min, observed by Hossein Janghorbani from the Astronomy and Geophysics Center of Shahreza at 1902.05 m of elevation. The new moon observation has arc of vision of 3.01°, with elongation of 8.23°, difference in azimuth of 7.66°, and width size of 10.27 arc sec. The smallest lag time observed by naked eye is 29.23 min, observed by John Caldwell from McDonald Observatory at 1954.67 m of elevation. The new moon observation has arc of vision of 5.72°, with elongation of 17.53°, difference in azimuth of 16.58°, and width size of 45.7 arc sec. The position of the data is portrayed in Fig. 7.

Moon age is the time in hours between sunset to moon conjunction for new moon observation, or moon conjunction to sunrise for old moon observation. The smallest moon age observed by observer is 5.98 h before sunset by Martin Elsaesser. The new moon was observed using CCD imaging. The arc of light of the moon is 6.52°, with elongation of 4.87°, arc of vision of 4.34°, difference in azimuth of 4.87°, and 6.47 arc sec width. For after sunset observation, the smallest moon age is 9.23 h

Table 2
World Records of Lunar Crescent Sighting.

No	Method of Observation	Records	Parameter	Elevation & Location	Date of Observation & Time Zone	Observer	Source
1	CCD	Smallest Arc of Vision: 0.46°	*LT*: 2 min *ArcL*: 15.21° *DAZ*:15.21° *W*: 32.36″	1100 m 49.07° N. 13.1° E,	2013/9/6 TZ: 2	Martin Elsaesser	https://www.astronomycenter.net/icop/kea34.html?l=en
2		Smallest Arc of Light = 4.67°	*LT*: 31.6 min *ArcV*: 4.65° *DAZ*: 0.35° *W*: 2.93″	518.06 m, 48.14° N, 11.58° E	2012/4/21 TZ: 2	Martin Elsaesser	https://www.mondatlas.de/other/martinel/sicheln2012/april/index.html
3		Smallest Lag Time: 2 min	*ArcV*: 0.46° *ArcL*: 15.21° *DAZ*: 15.21° *W*: 32.36″	1100 m, 49.07° N. 13.1° E,	2013/9/6 TZ: 2	Martin Elsaesser	https://www.astronomycenter.net/icop/kea34.html?l=en
4		Smallest Moon Age: 9.23 h	*ArcV*: 5.92° *ArcL*: 6.16° *DAZ*: 1.71° *W*: 5.58″	518.06 m, 48.14° N, 11.58° E	2011/9/27 TZ: 2	Martin Elsaesser	https://www.astronomycenter.net/icop/ram34.html?l=en
5		Smallest Width: 2.93″	*ArcV*: 4.65° *ArcL*: 4.67° *DAZ*: 0.35° *LT*: 31.6 min	518.06 m, 48.14° N, 11.58° E	2012/4/21 TZ: 2	Martin Elsaesser	https://www.mondatlas.de/other/martinel/sicheln2012/april/index.html
6	Telescope	Smallest Arc of Vision: 1.29°	*LT*: 7.89 min *ArcL*: 21.85° *DAZ*: 21.82° *W*: 64.5″	1100 m 49.07° N. 13.1° E,	2012/7/19 TZ: 2	Martin Elsaesser	https://www.astronomycenter.net/icop/ram42.html?l=en
7		Smallest Arc of Light: 5.98°	*LT*: 24.04 min ArcV: 4.85° *DAZ*: 3.5° *W*: 4.86″	722.48 m, 32.22° N, 110.97° W	2012/3/22 TZ: −7	Jim Stamm	https://www.astronomycenter.net/icop/jua33.html?l=en
8		Smallest Lag Time: 13.7 min	*ArcL*: 9.64° *ArcV*: 2.63° *DAZ*: 9.27° *W*: 12.55″	722.48 m, 32.22° N, 110.97° W	2014/6/27 TZ: −7	Jim Stamm	https://www.astronomycenter.net/icop/ram35.html?l=en
9		Smallest Moon Age: 11.09 h	*ArcL*: 5.98° *ArcV*: 4.85° *DAZ*: 3.5° *W*: 24.04″	722.48 m, 32.22° N, 110.97° W	2012/3/22 TZ: −7	Jim Stamm	https://www.astronomycenter.net/icop/ram35.html?l=en
10		Smallest Width: 4.86″	*ArcL*: 5.98° *ArcV*: 4.85° *DAZ*: 3.5° *LT*: 24.04 min	722.48 m, 32.22° N, 110.97° W	2012/3/22 TZ: −7	Jim Stamm	https://www.astronomycenter.net/icop/jua33.html?l=en
11	Optical Aided without Digital Imaging	Smallest Arc of Vision: 3.01°	*LT*: 16.05 min *ArcL*: 8.23° *DAZ*: 7.66° *W*: 10.27″	1902.05 m, 31.9° N, 51.87° E	2017/6/24 TZ: 3.5	Hossein Janghorbani	https://www.astronomycenter.net/icop/kea31.html?l=en
12		Smallest Arc of Light: 7.45°	*LT*: 38.23 min *ArcV*: 6.99° *DAZ*: 2.58° *W*: 8.4″	1575.56 m, 32.67° N, 51.67° E	2013/12/3 TZ: 3.5	Alireza Mehrani	https://www.astronomycenter.net/icop/saf35.html?l=en
13		Smallest Lag Time: 16.05 min	*ArcL*: 8.23° *ArcV*: 3.01° *DAZ*: 7.66° *W*: 10.27 ″	1902.05 m, 31.9° N, 51.87° E	2017/6/24 TZ: 3.5	Hossein Janghorbani	https://www.astronomycenter.net/icop/shw38.html?l=en
14		Smallest Moon Age: 13.02 h	*ArcL*: 8.13° *ArcV*: 8.1° *DAZ*: 0.75° *W*: 10.12″	1954.67 m, 30.67° N, 104.02° W	2014/1/1 TZ: −5	John Caldwell	(Caldwell, 2011)
15		Smallest Width: 8.4″	*ArcL*: 7.45° *ArcV*: 6.99° *DAZ*: 2.58° *LT*: 38.23″	1575.56 m, 32.67 N, 51.67 E	2013/12/3 TZ: 3.5	Alireza Mehrani	https://www.astronomycenter.net/icop/saf35.html?l=en
16	Naked Eye Observation	Smallest Arc of Vision: 5.3°	*LT*: 27.87 min *ArcL*: 22.37° *DAZ*: 21.74° *W*: 74.94″	634.3 m, 37.23° N, 80.41° W	2010/9/9 TZ: −4	Javad Torabinejad	https://www.astronomycenter.net/icop/shw31.html?l=en
17		Smallest Arc of Light: 7.45°	*LT*: 38.57 min *ArcV*: 7.44°	1185.77 m, 35.6° N, 83.5° W	1990/2/25 TZ: −4	John Pierce	(Doggett et al., 1994)

(*continued on next page*)

Table 2 (*continued*)

No	Method of Observation	Records	Parameter	Elevation & Location	Date of Observation & Time Zone	Observer	Source
			DAZ: 0.52° *W*: 8.22″				
18		Smallest Lag Time: 29.23 min	*ArcL*: 17.53° *ArcV*: 5.72° *DAZ*: 16.58° *W*: 45.7″	1954.67 m, 30.67° N, 104.02° W	2010/10/8 TZ: −5	John Caldwell	(Caldwell, 2011)
19		Smallest Moon Age: 14.63 h	*ArcL*: 7.45° *ArcV*: 7.44° *DAZ*: 0.52° *W*: 8.22″	1185.77 m, 35.6° N, 83.5° W	1990/2/25	John Pierce	(Doggett et al., 1994)
20		Smallest Width: 8.98″	*ArcL*: 8.07° *ArcV*: 8.07° *DAZ*:0.29° : 44.06	26.45 m, 32.47° N, 44.55° E	−154/5/15 TZ: 3	Babylonian	(Krauss, 2012)

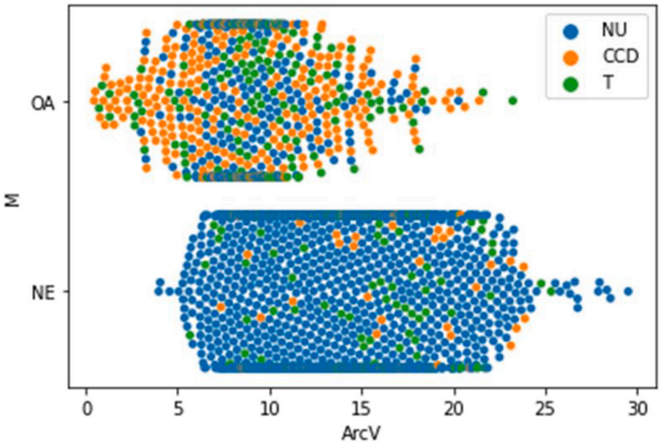

Fig. 5. Swarm Plot of Arc of Vision.

by Martin Elsaesser using CCD. The arc of light of the moon is 6.16°, arc of vision of 5.92°, difference in azimuth of 1.71°, and 5.58 arc sec width. The smallest moon age observed by telescope is 11.09 h, observed by Jim Stamm. The new moon observation has 4.85 degrees of arc of vision, 3.5 degrees of difference in azimuth, and 4.86 arc sec of width. The smallest moon age observed by binoculars is 13.02 h, observed by John Caldwell. The new moon observation has 8.1 degrees of arc of vision, 8.13 degrees of arc of vision, 0.75 degree of difference in azimuth, and 10.12 arc sec of width. The smallest naked eye observation for moon age is 14.63° by John Pierce, with arc of vision of 7.44°, difference in azimuth of 0.52°, and width size of 8.22 arc sec, with lag time of 38.57 min.

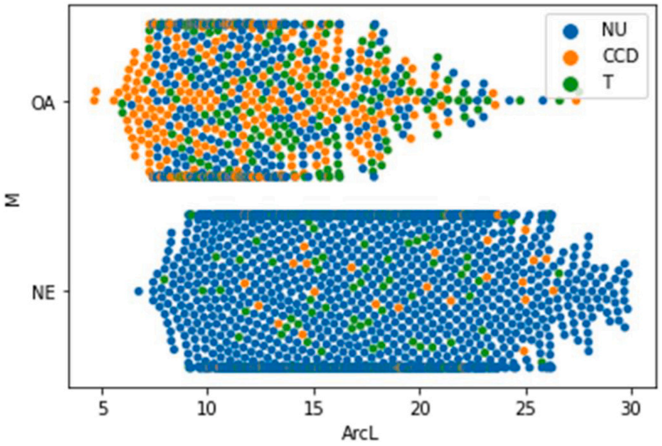

Fig. 6. Swarm Plot of Arc of Light.

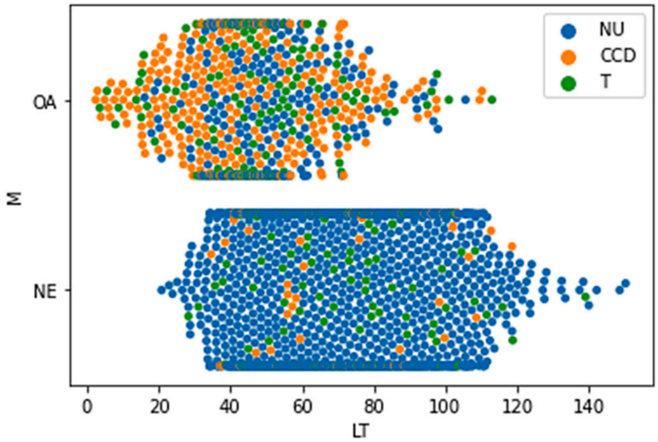

Fig. 7. Swarm Plot of Lag Time.

The position of the data is portrayed in Fig. 8.

Width is an illuminated area of the Moon measured along the Moon's diameter. The smallest width observed by CCD is 2.93 arc sec width by Martin Elsaesser. The lag time of the moon is only 31.6 min, with arc of vision of 4.65°, difference in azimuth of 0.35°, and 4.67 degrees of arc of light. The smallest width observed by telescope is 4.86 arc sec, observed by Jim Stamm. The new moon observation has 4.85 degrees of arc of vision, 3.5 degrees of difference in azimuth, and 5.98 degrees of elongation. The smallest width observed by binoculars is 8.4 arc sec, observed by Alireza Mehrani at 2080 m above sea level. The new moon observation has 6.99 degrees of arc of vision, 2.58 degrees of difference

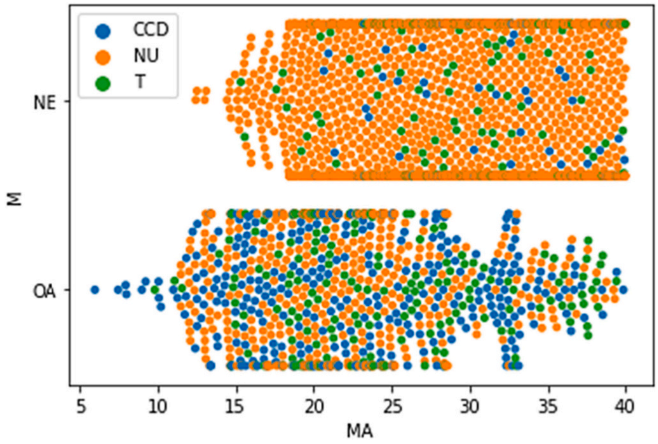

Fig. 8. Swarm Plot of Moon Age.

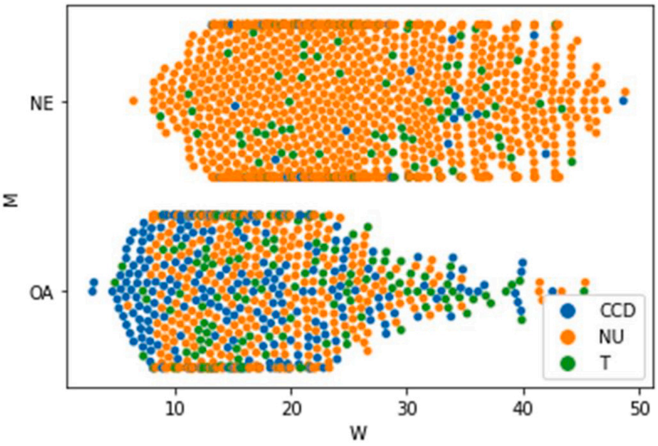

Fig. 9. Swarm Plot of Width.

in azimuth, and 7.45 degrees of elongation. The position of the data is portrayed in Fig. 9.

3.3. Arc of vision over elongation lunar crescent visibility criterion

Fig. 10 demonstrates position of lunar crescent visibility criteria of Istanbul 2016, MABIMS 1995, and MABIMS 2021. Istanbul 2016, MABIMS 1995 and MABIMS 2021 interestingly share the same nature of their contradiction rate. These criteria have a low negative contradiction rate, where Istanbul has 15.61%, MABIMS 1995 having 6.85 and MABIMS 2021 having 6.82%. These criteria are designed to reduce any positive lunar crescent sighting locates below their visibility line. This is common for lunar crescent visibility criterion that adapted for calendrical purposes, as it is to avoid error in positive sighting. MABIMS lunar crescent visibility criterion which adapted by Brunei, Indonesia, Malaysia, and Singapore for their Islamic calendar regulation cannot have any single positive lunar crescent sighting that fall below its visibility line. A single positive lunar crescent observation that falls below the criterion visibility will lead to confusion in determining the exact date of new Hijri month. Istanbul 2016 functioned similarly with MABIMS lunar crescent visibility criterion to avoid sighted lunar crescent below its visibility line. The contradiction rate of the criteria is portrayed in Table 3.

Table 3
Arc of Vision Vs Arc of Light Lunar Crescent Visibility Criteria.

	ISTANBUL 2016		MABIMS 1995		MABIMS 2021	
	Positive	Negative	Positive	Negative	Positive	Negative
Whole	29.04	15.61	37.29	6.85	33.49	6.82
Naked Eye	34.2	2.74	43.27	0.0	39.22	0.18
Optical Aided	0.94	100.0	0.90	100.0	0.91	100.0

Another trait of criterion that is designed for calendrical purposes is that it favors naked eye observation rather than optical aided observation. This is evidenced in MABIMS 1995 and MABIMS 2021, having smaller negative naked eye sighting contradiction rate, while Istanbul has only 2.74% positive naked eye sighting contradiction rate. The rationale behind prioritizing naked-eye observation for lunar crescents is rooted in the fact that optical-aided observation is typically carried out by experts and is not easily accessible to the general public. Naked-eye observation, in contrast, can be performed by the masses and is accessible to people across different segments of the population. A positive naked-eye observation below the visibility limit tends to attract attention and may lead to more confusion among the public. Conversely, a positive optical-aided observation below the visibility limit generally garners interest among experts but generates less widespread attention compared to naked-eye observation.

Istanbul 2016 lunar crescent visibility criterion is not effective as a lunar crescent visibility criterion, as there are numbers of naked eye lunar crescent sighting that sits below the criterion visibility line. A naked eye lunar crescent sighting that goes against a lunar crescent visibility criterion would create confusion in the public. MABIMS 1995 while effective at negating any contradicting positive naked eye and optical aided observation, are too close to horizon and does not reflect an actual observation of lunar crescent. A Hijri calendar purposed lunar

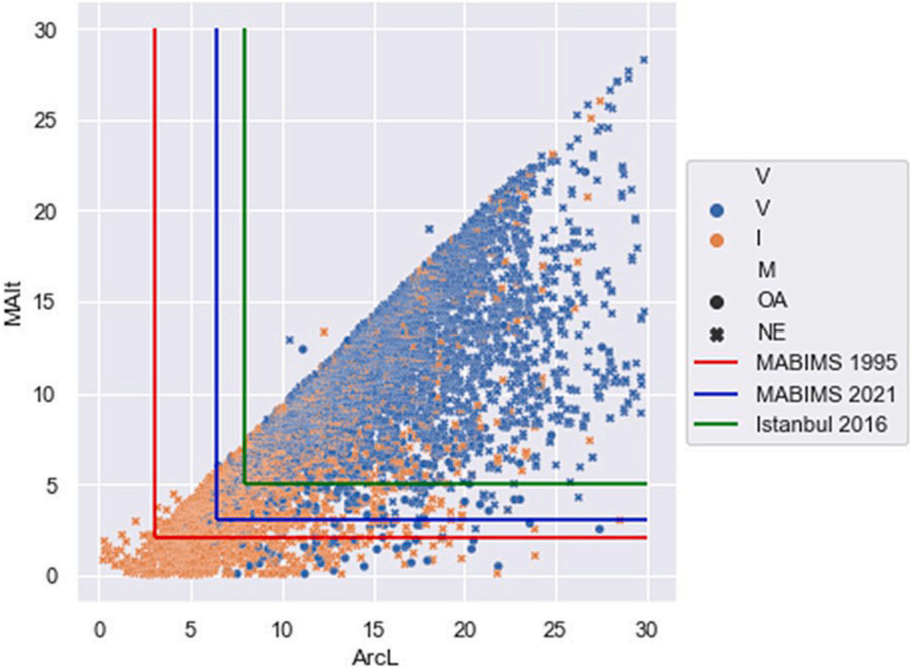

Fig. 10. Scatter Plot for Moon Altitude Over Arc of Light Lunar Crescent Visibility Criterion.

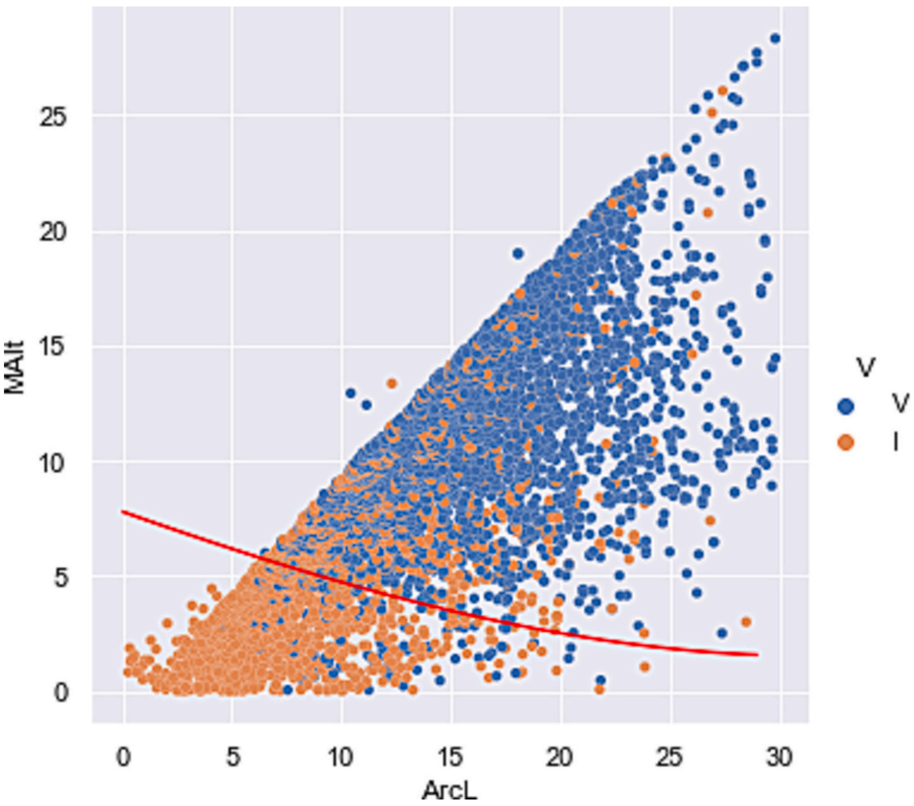

Fig. 11. Scatter Plot Suggested Lunar Crescent Visibility Criterion.

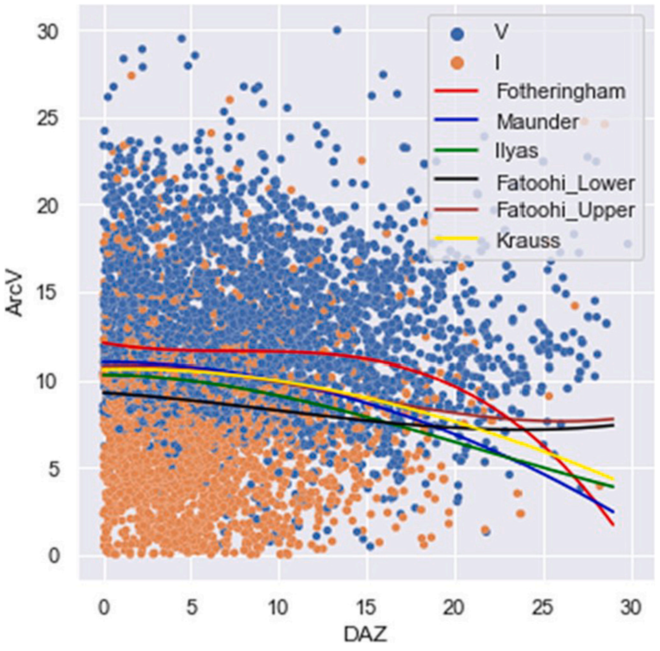

Fig. 12. Scatter Plot for Arc of Vision Over Difference in Azimuth Lunar Crescent Visibility Criterion.

crescent visibility criterion must reflect an actual observation of lunar crescent during 29th of an Islamic month. MABIMS 2021 is the most suitable criterion for Hijri calendrical purpose. The first reason is that MABIMS 2021 lunar crescent visibility criterion reflects an actual observation of lunar crescent, by having a parameter based on lower limit of naked eye observation of lunar crescent. Second, MABIMS 2021 does not have any positive naked eye lunar crescent sighting that is located below its criterion line.

However, there are number of recommendations to improve MABIMS 2021 as a Hijri calendrical reference. First, MABIMS 2021 6.4° elongation parameter is closely located above the minimum telescopic records of optical aided observation, which is 5.96°. Currently there are two telescopic lunar crescent observations that are located below 6.4° elongation limit. Pairs with high value of arc of vision and lunar crescent width, a skillful observer able to break the 6.4° elongation parameter limit. As telescopic observation is now becoming norm sighting methodology among religious officials, observatories, and amateurs astronomy, a lunar crescent visibility criterion should have elongation parameter close to minimum telescopic observation records, in accordance with the lunar crescent observation norm. Should MABIMS 2021 adopted telescopic observation as criterion reference, a value of 5.50 degree of elongation is recommended, as it located between minimum visibility telescopic observation and CCD observation, which are 5.96° and 4.63° subsequently.

Second, MABIMS 2021 lunar crescent visibility criterion parameter design is not dynamic and unable to follow the changes nature of lunar

Table 4
: Arc of Vision Vs Difference in Azimuth Lunar Crescent Visibility Criteria.

	Fotheringham		Maunder		Ilyas		$Fatoohi_{LL}$,		$Fatoohi_{UL}$,		Krauss	
	(+)	(−)	(+)	(−)	(+)	(−)	(+)	(−)	(+)	(−)	(+)	(−)
Whole	11.46	46.01	13.09	38.16	15.09	33.31	17.28	25.67	13.24	37.93	13.43	37.13
Naked Eye	12.03	33.11	14.05	22.17	16.37	15.71	19.29	8.58	14.22	21.82	14.46	20.76
Optical Aided	0.0	98.96	0.0	98.81	1.08	99.19	1.4	99.63	0.0	98.8	0.0	98.78

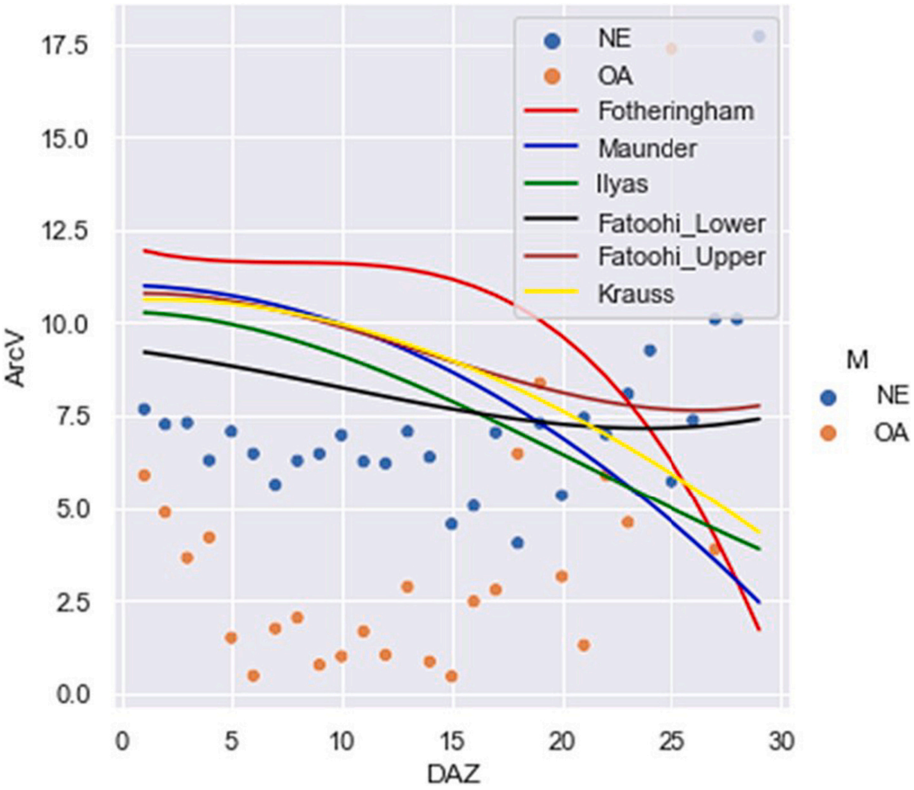

Fig. 13. Position of the criteria over minimum data of lunar crescent sighting.

Table 5
Regression Analysis for Arc of Vision Vs Difference in Azimuth Lunar Crescent Visibility Criteria.

Criterion	Fotheringham	Maunder	Ilyas	*Fatoohi*$_{LL}$,	*Fatoohi*$_{UL}$,	Krauss
Mean Absolute Error	6.13	4.83	4.23	3.55	4.62	4.71
Mean Squared Error	48.5	33.95	27.11	19.57	29.38	31.6
R Squared	−3.02	−1.81	−1.25	−0.62	−1.44	−1.62

crescent cycle. The way the MABIMS 2021 designed is that it follows are two logic condition, which are arc of vision condition and arc of light conditions. This kind of design has its flaws, as there will be cases where a sighted lunar crescent has an arc of vision condition that passes MABIMS 2021 and does not pass MABIMS 2021 arc of elongation condition. Alternatively, there will be cases where a sighted lunar crescent has an arc of light higher than MABIMS 2021 parameter but does not pass MABIMS 2021 arc of vision parameter. These cases are plausible for location with high latitude as elongation value is higher than location that located near the equator. In our data, there are 65 positive lunar crescent observations that do not pass one of MABIMS 2021 criteria, and 3 of the observations are located at Southeast Asia.

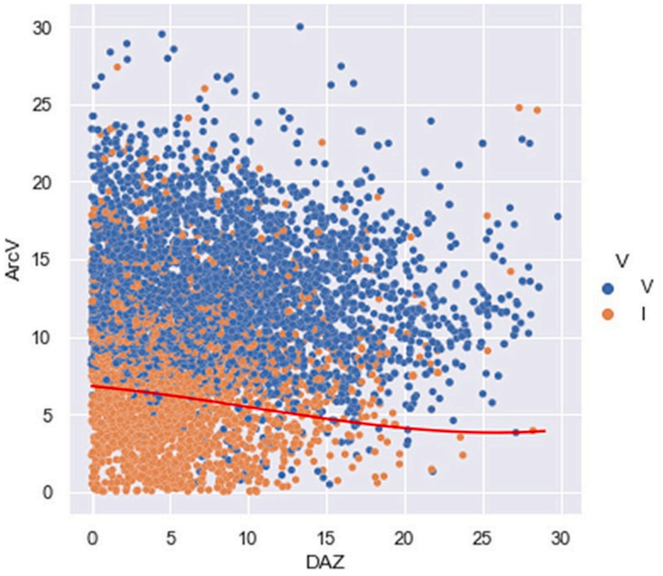

Fig. 14. Suggested Criterion for Naked Eye Sighting.

To negate this issue, a calendrical lunar crescent visibility criterion should utilize an expression as its design parameters. Expression parameters are more dynamic, and able to composite both arc of vision and arc of light into a single condition. This reduces the chances of sighted lunar crescent observation that located below the criterion line, particularly for high latitude location. As a suggestion, moon altitude versus arc of light lunar crescent visibility criterion that prioritize removal of negative naked eye sighting contradiction is expressed as Eq. 1.

$$MAlt = -0.3351\,ArcL + 0.0023\,ArcL^2 + 0.000064\,ArcL^3 + 7.78 \tag{1}$$

The suggested lunar crescent visibility criterion is able to predict visibility of lunar crescent with 30.41% positive contradiction rate and 0.0% negative contradiction rate for naked eye prediction. This is better than MABIMS 2021, MABIMS 1995, and Istanbul 2021 in predicting lunar crescent visibility and applicable for various range of location latitude. The position of the lunar crescent visibility criterion over data of lunar crescent sighting is portrayed in Fig. 11.

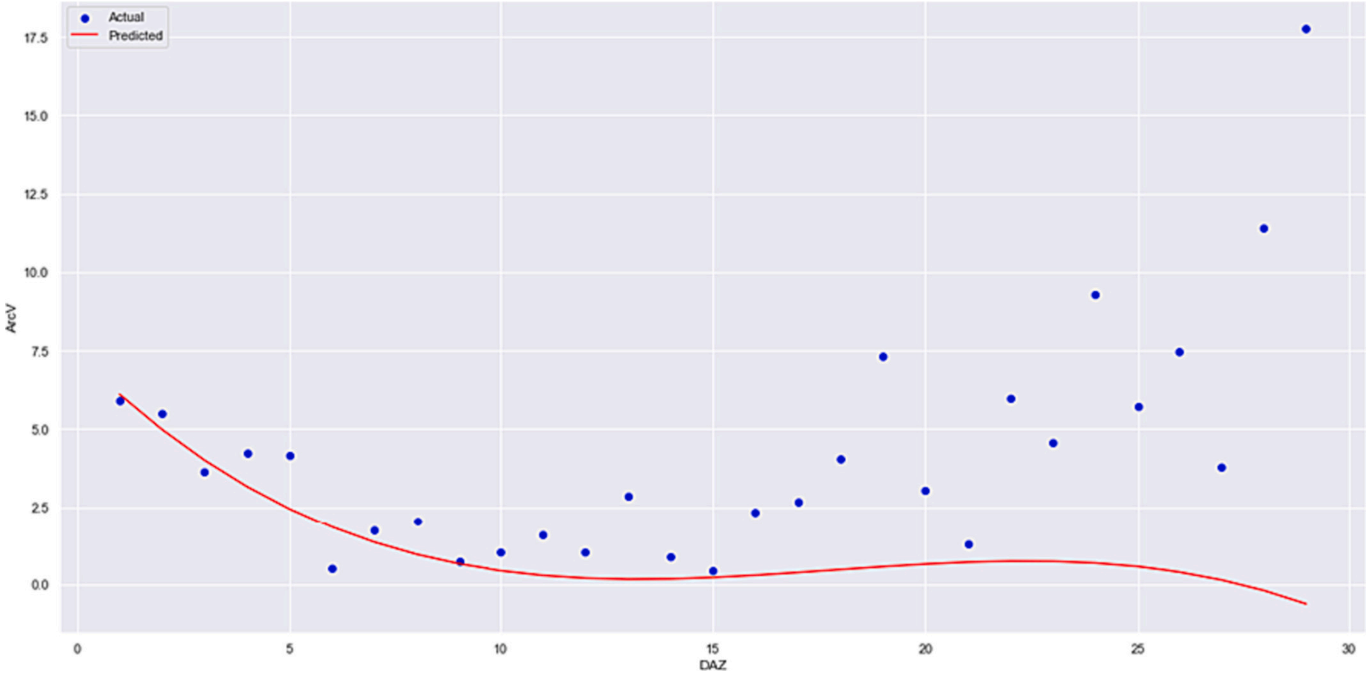

Fig. 15. Suggested Criterion for Optical Aided Sighting.

3.4. *Arc of vision over azimuth lunar crescent visibility criterion*

Arc of vision versus azimuth lunar crescent visibility criterion is introduced by Fotheringham, (1910). Fotheringham does not state where does he gather ideas to construct the lunar crescent visibility curve using altitude and azimuth parameter, although Ilyas (1987) claimed that it was inspired by Battani lunar crescent visibility curve. Maunder (1911) published an improvement over Fotheringham, then followed by Ilyas (1988), (Fatoohi, 1998) and (Krauss, 2012). The position of their criteria over data of lunar crescent sighting is portrayed in Fig. 12.

Ilyas lunar crescent visibility criterion, which criticized as underestimate human eye capabilities in detecting lunar crescent (Schaefer, 1991a, 1991b), is actually located lower than most lunar crescent visibility criterion. Most arc of vision over arc of light lunar crescent visibility criterion is located higher, with Fotheringham at the highest line. Consequently, these criteria are able to predict lunar crescent that located above the criterion accurately, however, weak in predicting lunar crescent visibility criterion that located below the criterion. As evidence, Fotheringham lunar crescent visibility criterion has lowest positive error rate contradiction, at 11.46%, and largest negative error rate contradiction at 46.01%. In contrast, lunar crescent visibility criterion that located at the lower line such as Fatoohi and Ilyas, is better at predicting negative lunar crescent sighting and worse at predicting positive lunar crescent sighting. As evidence, Fatoohi, has 25.67% negative error rate contradiction, and 17.28% at positive contradiction

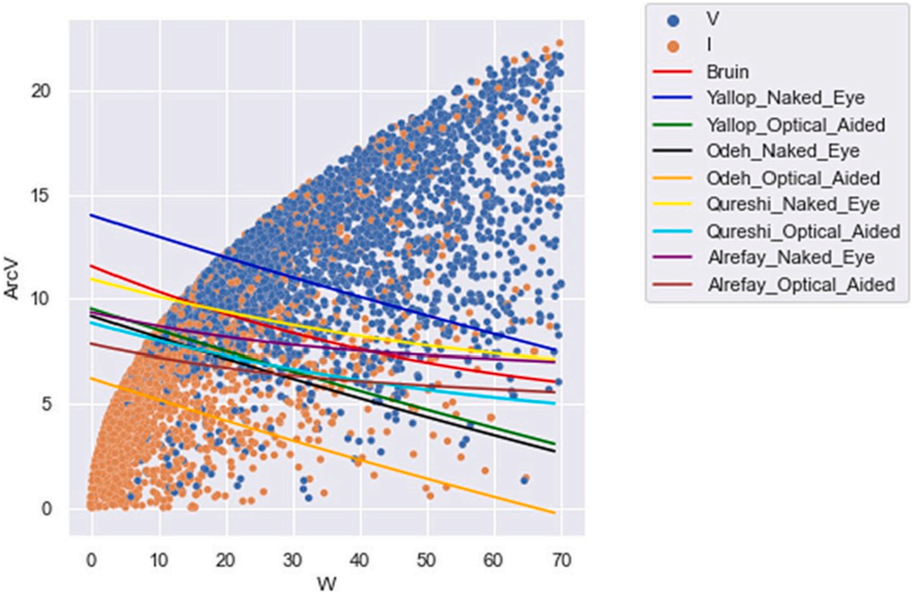

Fig. 16. Scatter Plot for Arc of Vision Over Width Lunar Crescent Visibility Criterion.

Table 6
ArcV vs W Contradiction Rate.

Criterion	Bruin		Yallop Naked Eye		Yallop Optical Aided		Odeh Naked Eye		Odeh Optical Aided		Qureshi Naked Eye		Qureshi Optical Aided		Alreday Naked Eye		Alreday Optical Aided	
	(+)	(−)	(+)	(−)	(+)	(−)	(+)	(−)	(+)	(−)	(+)	(−)	(+)	(−)	(+)	(−)	(+)	(−)
Whole	16.77	29.93	12.21	43.3	16.6	31.92	21.6	15.79	30.02	5.15	16.75	31.06	21.71	16.49	19.43	22.22	23.89	12.7
Naked Eye	18.64	12.46	13	29.51	18.33	14.69	25.23	2.23	35.79	0	18.52	13.53	25.36	3.05	22.28	6.55	28.28	2.07
Optical Aided	0.93	99.08	0	98.91	1.01	99.13	1.28	100	0.98	100	0.99	99.12	1.29	100	1.4	99.77	1.14	100

rate. The full report of the criteria contradiction rate is portrayed in Table 4.

Fig. 13 demonstrates the position of the criteria over minimum data of lunar crescent sighting. All the arc of vision versus difference in azimuth lunar crescent visibility criteria ignores optical aided observation of lunar crescent sighting. This is because data of lunar crescent sighting in the database of Fotheringham, Maunder, Ilyas, Fatoohi, dan Kraus do not have optical aided lunar crescent sighting, particularly Telescopic, and CCD sighting. The usage of telescope and CCD in the lunar crescent observation only start primarily in the early 21st century. Due to the ignorance of optical aided lunar crescent sighting, most of the arc of vision versus difference in azimuth criteria has high value of Mean Absolute Error (MAE) and Mean Squared Error (MSE). This is demonstrated in Table 5. As a suggestion, arc of vision versus difference in azimuth lunar crescent visibility criterion that prioritize removal of naked eye negative sighting contradiction is expressed as Eq. 2.

$$ArcV = -0.09222\,DAZ - 0.00629\,DAZ^2 + 0.0002078\,DAZ^3 + 6.792 \tag{2}$$

The suggested lunar crescent visibility criterion is able to predict visibility of lunar crescent with 29.85% positive contradiction rate and 0.43% negative contradiction rate for naked eye prediction. This is better in predicting naked eye lunar crescent visibility and applicable for various range of location latitude. The position of the lunar crescent visibility criterion over data of lunar crescent sighting is portrayed in Fig. 14.

For criterion that prioritize in locating around the optical aided sighting is expressed as Eq. 3.

$$ArcV = -1.3590\,DAZ + 0.081710\,DAZ^2 - 0.0015330\,DAZ^3 + 7.391 \tag{3}$$

The lunar crescent visibility criterion is better at predicting optical aided lunar crescent sighting, and it is located around the data of optical aided sighting. The criterion mean absolute error of the criterion is 3.25 and mean squared error is 26.55, value better than most arc of vision over difference in azimuth lunar crescent visibility criterion. The shape of the criterion is portrayed in Fig. 15.

3.5. .Arc of vision over width lunar crescent visibility criterion

Arc of vision versus width lunar crescent visibility criterion is introduced by Bruin (1977). Bruin incorporates the parameter of lunar width, altitude, and azimuth in his criterion. The application of Bruin lunar crescent visibility criterion is complicated. First the width of the observed lunar crescent needs to be calculated first. Taking example lunar crescent width of 2′, during lunar crescent observation, at 5.5 degrees of lunar altitude, lunar crescent is visible at solar depression of 4.0° until 0.8°, meaning that it has 12.8 min windows of opportunity. Bruin lunar crescent visibility criterion not only able to predict the visibility of the lunar crescent is at the same time able to estimate the time windows for successful observation. Bruin in designing his criterion, has the following assumption. First, he assumes that the sky brightness is uniform regardless of altitude and azimuth, with only solar depression as a single brightness variable as he located from findings of Koomen et al. (1952) and Tousey and Koomen (1953). Second, Bruin assumes that the brightness of the lunar crescent is uniform all across its surface, with only lunar crescent altitude acts as a presenter for atmospheric extinction from Bemporad (1904) research. Third, Bruin assumes that minimum required contrasts for lunar crescent visibility are associated with lunar surface area. For this assumption Bruin adopted the work of Siedentopf (1940) circular disk visibility threshold and converted it into lunar width. Bruin uses assumption in his design for lunar crescent visibility criterion, so he does not use based on actual observation of lunar crescent. Bruin stated that his criterion has been experiment for 10 years, and his assumption are correct without requiring further refinement.

All of the three Bruin assumptions could be improved (Faid et al.,

Table 7
ArcV vs W Regression Analysis.

Criterion	Bruin	Yallop Naked Eye	Yallop Optical Aided	Odeh Naked Eye	Odeh Optical Aided	Qureshi Naked Eye	Qureshi Optical Aided	Alrefay Naked Eye
Mean Absolute Error (MAE)	3.5	5.12	3.73	3.37	4.59	3.49	2.85	2.59
Mean Squared Error (MSE)	21.32	37.69	23.4	20.62	33.23	20.59	15.53	13.27
R Squared	−0.92	−2.4	−1.11	−0.86	−1.99	−0.86	−0.4	−0.2

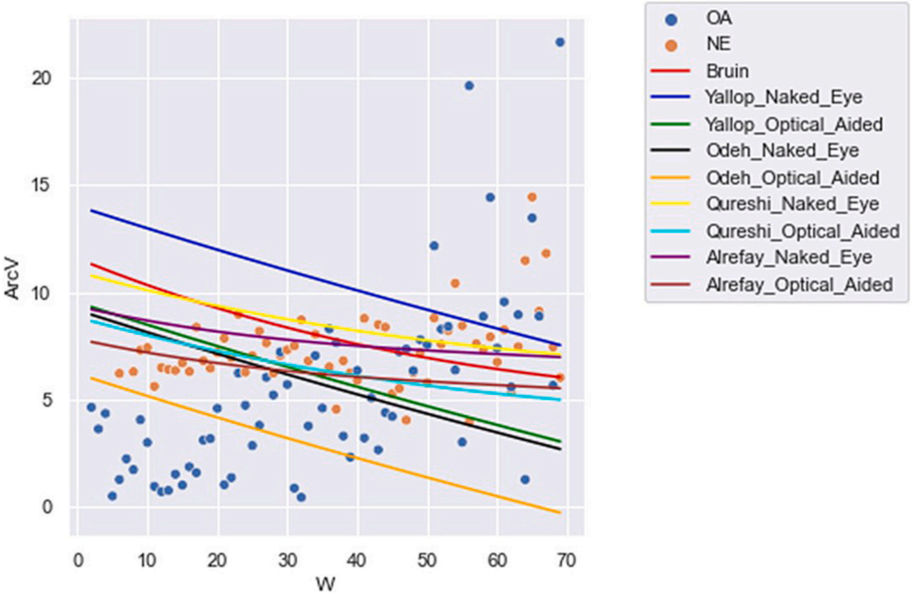

Fig. 17. Scatter Plot of Minimum Data for Arc of Vision Over Width Lunar Crescent Visibility Criterion.

2022). First, the assumption that sky brightness is uniform will single solar depression act a brightness variable does not entirely represent the actual observation. Kastner (1976) has developed a modelling that demonstrate the brightness of sky during twilight are dependent to solar depression, altitude, and azimuth of the observed object. Kastner modelling warrant a high accuracy and still relevant for modern application (Crumey, 2014; Faid et al., 2023c). Second, the assumption that brightness of the lunar crescent to be singularity dependent to lunar crescent altitude could be improved. Although lunar crescent altitude can represent atmospheric extinction in the simplest form, the impact of atmospheric extinction on lunar brightness is more complex and requires complex variables. Schaefer (2000) has laid out computation required to measure the impact of atmospheric extinction to lunar brightness, encompassing air mass, temperature, season, atmospheric layer, humidity, altitude, latitude, and wavelength. Thus, to simply express impact of atmospheric extinction on lunar brightness in the form of lunar crescent altitude is an oversimplification. Third, Bruin adopted Siedentopf circular disk visibility threshold in his criterion by assuming its applicable for lunar crescent visibility threshold. Circular disk visibility and lunar crescent visibility are heterogenous. This is because the surface area and the shape of lunar crescent is entirely different with circular disk. Blackwell (1946) in model of visibility threshold in 1946, are more suitable for Bruin lunar crescent visibility criterion instead of Siedentopf works. Despite Bruin imperfect assumption, he is the first to introduce width in modern lunar crescent visibility criterion. Bruin work is then followed by Ilyas (1988); Odeh (2004); Qureshi (2010) and Alrefay et al. (2018). The position of their criteria over data of lunar crescent sighting is portrayed in Fig. 16.

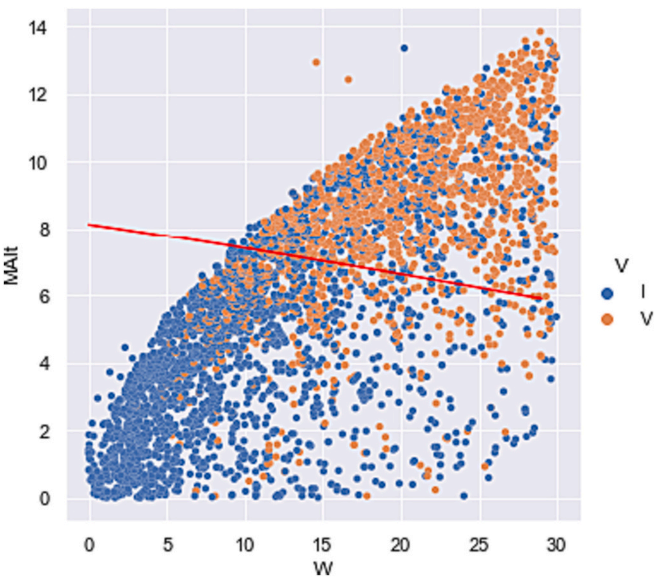

Fig. 18. Recommended Criterion for Naked Eye Sighting.

Odeh follows Yallop's model of lunar crescent sighting, therefore both have the same line with different y-axis starting point. Qureshi and Alrefay add some additional curve in their criterion line, however it still has similar shape with Yallop and Odeh criteria. In predicting naked eye sighting, Odeh Naked Eye criterion is the best naked eye lunar crescent visibility criterion, with 1positive contradiction rate of 25.23 %, and

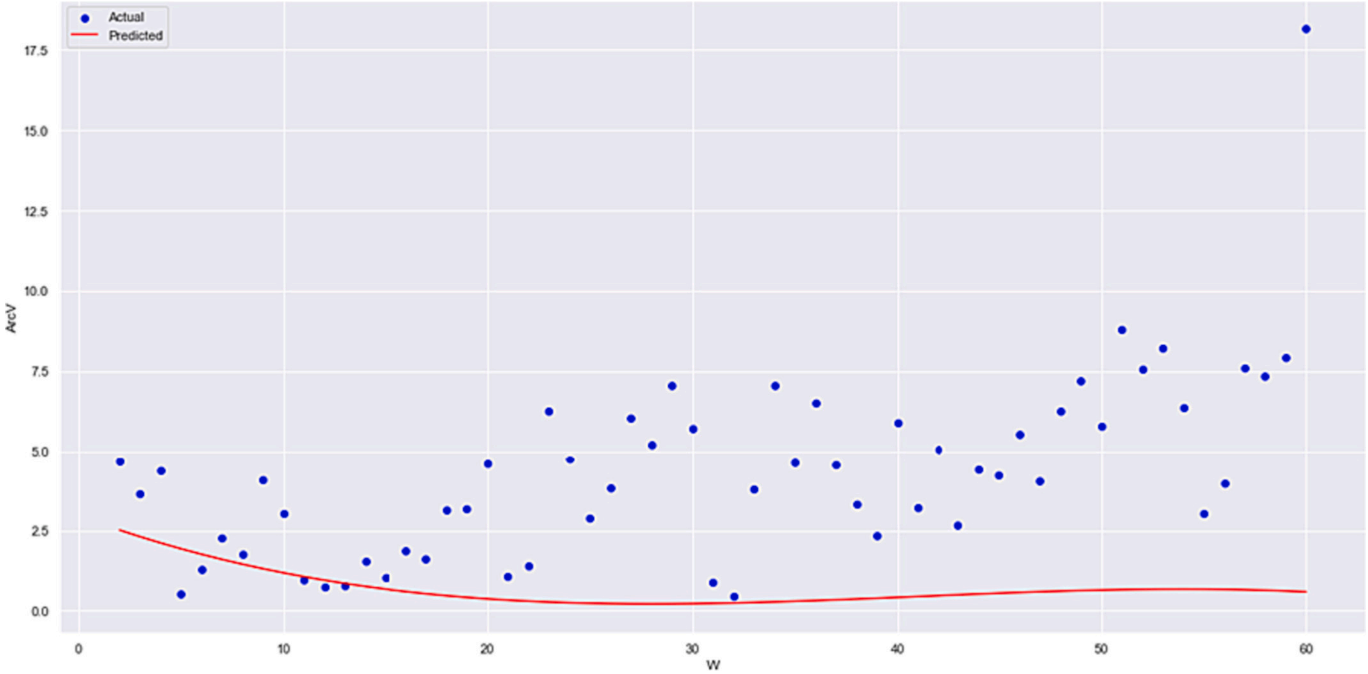

Fig. 19. Recommended Criterion Over Minimum Optical Aided Sighting.

negative contradiction rate of 2.23 %. Yallop Naked Eye criterion, however, is worse at predicting naked eye sighting, with an positive contradiction rate of 13, and negative contradiction rate of 29.51. Most of the lunar crescent visibility criterion located above the majority optical aided lunar crescent sighting, except for Odeh Optical Aided lunar crescent visibility criterion, where it located around the optical aided sighting. However, there are a number of lunar crescent sightings that observed at lower parameter of arc of vision and width, making most of these type of criterion unable to predict negative lunar crescent visibility accurately. The assesment result can be located here in Tables 6 and 7. The criterion position over minimum value of lunar crescent sighting is portrayed in Fig. 17.

In contrast to criteria that use elongation and arc of vision, using parameter of width is not appropriate for predicting lunar crescent visibility, especially lunar crescent observations that located at extreme value. As a suggestion, arc of vision versus width lunar crescent visibility criterion that prioritize removal of naked eye negative sighting contradiction is expressed as Eq. 4.

$$ArcV = -0.2387791499\ W + 0.0053517999\ W^2 - 0.0000422340\ W^3 + 7.9662653619 \tag{4}$$

The suggested lunar crescent visibility criterion is able to predict visibility of lunar crescent with 34.31% positive contradiction rate and 0.29% negative contradiction rate for naked eye prediction. This is better in predicting naked eye lunar crescent visibility and applicable for various range of location latitude, with only 4 positive naked eye lunar crescent sighting located below the criterion. The position of the lunar crescent visibility criterion over data of lunar crescent sighting is portrayed in Fig. 18.

For optical aided sighting, criterion that prioritize in locating around the optical aided sighting is expressed as Eq. 5:

$$ArcV = -0.2372\ W + 0.0064324\ W^2 - 0.0000523\ W^3 + 2.957 \tag{5}$$

The lunar crescent visibility criterion is better at predicting optical aided lunar crescent sighting, and it is located around the data of optical aided sighting. The criterion mean absolute error of the criterion is 3.73 and mean squared error is 22.49, value better than most arc of vision over width lunar crescent visibility criterion for optical aided observation. The shape of the criterion is portrayed in Fig. 19.

3.6. *Lag time Lunar Crescent Visibility Criterion*

Generally, lag time is considered as weak parameter for a lunar crescent sighting. Schaefer (1996) highlighted that lag time has consistently high error throughout parameter and notes that lag time has is less predictable at high latitude. Ahmad et al. (2020) and Ilyas (1983) argue that lag time is only suitable for explaining visibility of lunar crescent for layman and not suitable for lunar crescent visibility criterion. Table 8 demonstrates the correlation result of lunar crescent visibility parameter. *LT*, *MA*, *ArcV*, *ArcL*, *DAZ*, *W*, and V are presented by lag time, moon age, arc of vision, arc of light, different in azimuth, width, and visibility respectively.

On visibility correlation, lag time and difference in azimuth has the weakest correlation with value of 0.19 and 0.20, respectively. This shows that lag time and difference in azimuth has the weakest correlation influence of lunar crescent visibility in comparison to other. This shows that it does not plays as a primary factor in determining visibility of a lunar crescent. Despite criticism of lag time parameter in lunar crescent visibility, there are scholars that adopted lag time in their criterion. Caldwell adopted lag time for his lunar crescent visibility

Table 8
Pearson Correlation Result of Lunar Crescent Visibility Parameter.

LT							
MA	0.93						
ArcV	0.22	0.2					
ArcL	0.2	0.28	0.72				
DAZ	0.11	0.23	0.074	0.7			
W	0.18	0.26	0.55	0.9	0.74		
V	0.19	0.18	0.53	0.5	0.2	0.35	
	LT	*MA*	*ArcV*	*ArcL*	*DAZ*	*W*	*V*

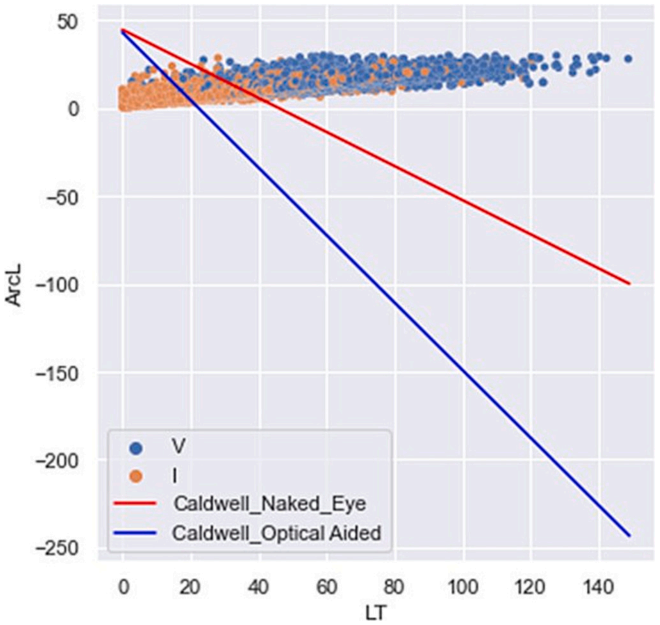

Fig. 20. Caldwell Lunar Crescent Visibility Criterion.

criterion (Caldwell. 2011). He argued that lag time is more suitable for various latitude of observation site. This is because while the arc of vision is relatively the same at various latitudes, lag time increases significantly at higher latitudes as the path of the lunar crescent is more slanted to the horizon. This increases the chance of visibility. Arc of vision unable to reproduce the slanted path of the lunar crescent, therefore it is not suitable for lunar crescent visibility criterion.

Fig. 20 demonstrates weakness of Caldwell lunar crescent visibility criterion. Caldwell lunar crescent visibility is limited to data of lunar crescent below 72 min for naked eye observation, and 20 lag time for optical aided observation. This does not represent the actual observation of lunar crescent, where lag time ranges from 2 min to over 140 min. The primary reason for Caldwell's weakness is due to the incompatibility of ranges between arc of light and lag time as a parameter for lunar crescent visibility criterion. The arc of light has minimum positive observation of 4.63°, and maximum of 45.14°, while lag time has minimum of 2.83 min, and 197.46 min. This huge difference in ranges cause Caldwell lunar crescent visibility criterion to be limited to data of lunar crescent below 72 min for naked eye observation, and 20 lag time for optical aided observation. Another reason is that arc of light distribution is similar with lag time distribution. This is because lag time is directly proportional to arc of light, lunar crescent that has higher lag time, usually has higher arc of light, and lunar crescent that has lower lag time usually has lower arc of light.

On the other hand, Gautschy (2014) able to produce a more robust lunar crescent visibility criterion using combination of lag time and difference in azimuth. While difference in azimuth has incompatible range with lag time, most of the Gautschy data of lunar crescent sighting has lower value of lag time, with maximum of 40 min. This negates the mismatch range issue between different in azimuth and lag time. In addition, lag time is not directly proportional to different in azimuth. A higher lag time does not necessarily translate into higher value of difference in azimuth. As the result, Gautschy lunar crescent visibility criterion line position is better than Caldwell lunar crescent visibility criterion, as portrayed in Fig. 21.

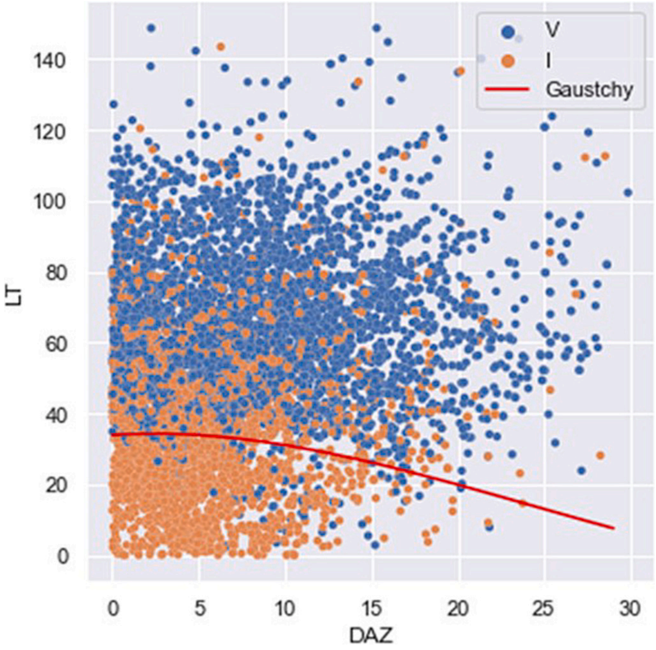

Fig. 21. Gautschy Lunar Crescent Visibility Criterion.

Table 9 demonstrated that Caldwell lunar crescent visibility criterion is weaker at predicting lunar crescent visibility in comparison to Gautschy criterion, Caldwell criterion line is located far from data of lunar crescent sighting, causing high mean square error. In comparison, Gautschy lunar crescent visibility criterion is located around the data of lunar crescent sighting, consequently, has lower mean square error, as illustrated in Table 10.

3.7. *Negative naked eye lunar crescent sighting outliers*

In theory lunar crescents that have high visibility parameter should

Table 9
Lag Time Lunar Crescent Visibility Criteria.

	Caldwell Naked Eye		Caldwell Optical Aided		Gautschy	
	Positive	Negative	Positive	Negative	Positive	Negative
Whole	22.48	11.17	32.12	4.72	24.67	8.82
Naked Eye	26.39	1.55	37.68	0.0	29.05	0.82
Optical Aided	1.08	100.0	0.94	100.0	1.02	100.0

Table 10
Regression Analysis of Lag Time Criteria.

Criterion	Caldwell Naked Eye	Caldwell Optical Aided	Gautschy
Mean Absolute Error (MAE)	38.98	99.36	16.66
Mean Squared Error (MSE)	2495.23	14,415.61	578.76
R Squared	−80.9	−472.14	−0.73

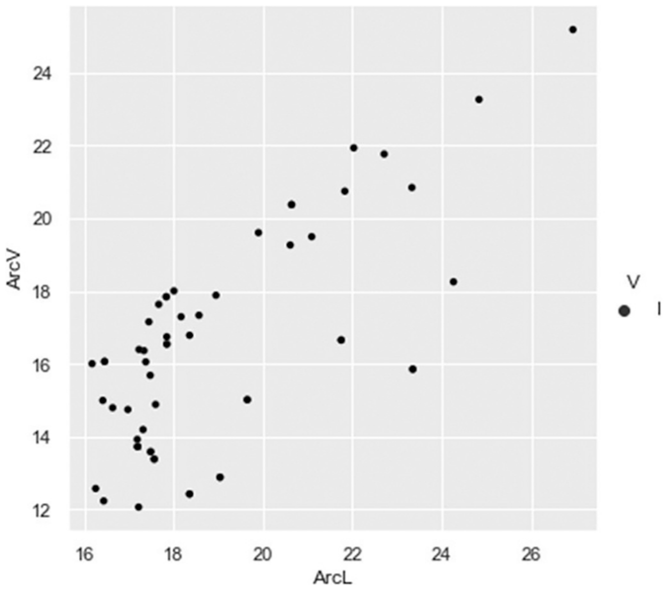

Fig. 22. Negative Lunar Crescent Sighting Outlier.

be able to be sighted by naked eye. However, there are cases where lunar crescent is not sighted, despite being situated high above the visibility line. These cases are called negative naked eye lunar crescent observation outlier. To define the threshold outlier might vary from one statistical data to another. Choosing outliers based on a specific criterion will bias the result in accordance with the specific criterion, making the definition of the outliers vary from on to another. In these cases, the outlier is selected based on two data, the lunar crescent observation correlation parameter, and its quartile location. This ensures the selection is free from bias and based on the actual scatter of the data.

Arc of vision and arc of light is selected as it has the highest correlation value in determining lunar crescent visibility based on Table 8. Their limiting value for the upper second quartile is 16° and 12° for arc of vision and arc of light, respectively. The assumption is that the lunar crescent is supposed to be sighted at lunar crescent above their upper second quartile parameter. The result of the filtering is 1268 data of lunar crescent observation, with 58 data of negative lunar crescent observation. The scattered plot of the negative lunar crescent observation is portrayed in Fig. 22.

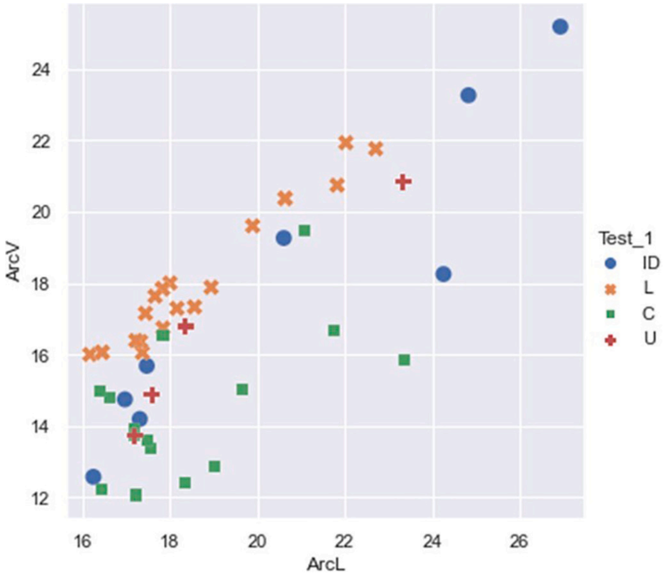

Fig. 23. Negative Lunar Crescent Sighting Outliers with Addition Test.

Addition test is conducted to further understand it outlier nature. The first test is the comparison with width parameter. If the lunar crescent has parameter of width below the upper first quartile of the parameter based on Fig. 9 and the lunar crescent is classified as ‘L’, signifying lower limit of the first quartile. The second test is to check dating accuracy of the data, if the date of the data is incorrectly reported, whether due to data date validity, which usually for the case of ancient data, data time zone and inconsistent from calculated Julian date and reported Julian data. The second test will be classified as “ID”, signifying incorrect data. The third test weather and atmospheric condition report. The International lunar crescent project report has information about the weather and atmospheric condition during the observation. The third test will be classified as ‘C’ signifying cloudy. Data that does not fall under the condition of these three tests is classified as “U”, signifying upper limit. The result of the additional test is as shown in Fig. 23.

There are nine data that classified as incorrect dating, from Krauss and Fatoohi’s Babylon data. Krauss conceded that Babylon recorded lunar crescent observation has 1 day contradiction rate, making it to be susceptible to being classified as outliers. 21 data located at lower limit from upper first quarter value of width and different in azimuth. Width is directly proportional to the surface area of the moon, while differences in azimuth correspond to the brightness of the background twilight sky. Thus, these 21 data are classified as outliers as is has low contrast value to be detected by human naked eye. 23 of the data is observed during cloudy weather and hazy atmospheric condition, making the lunar crescent not visible during observation. There are five data that is not sighted, despite locating above all parameter visibility line and having cloudless sky condition.

Schaefer has highlighted that eyesight, experience and age contribute to the probability of successful sighting, although the effect is small, it is non-neglectable and presents a real impact of sighting probability. Schaefer also added that error in locating the moon position, contribute to at least 2% from total collected data Doggett et al. (1994). In this case, all of the observers are members of ICOP and are well known experts in lunar crescent visibility among their community. However, as mentioned by Schaefer, this observation could fall into the categorization of 2% of human error from total collected data.

4. Conclusion

The assessment of lunar crescent visibility criterion is vital for Hijri calendar determination. Ilyas, Schaefer and Fatoohi has previously conducted research on this matter to provide insight on the reliability of a criterion. However, most of their researches are predated 1998 and require a refresh view since there are more data and visibility criterion since their published analysis. To reach the endeavor, this paper aims to provide a comparative analysis of lunar crescent visibility criterion, based on 8290 records, including 5267 positive lunar crescent sightings and 3023 negative records. The analysis is conducted using swarm plot analysis, contradiction rate analysis, and regression analysis. Analysis on arc of vision versus elongation lunar crescent visibility criterion, which favorably used in most of the Islamic countries, found that it designed to eliminate confusion in a naked-eye observation of lunar crescent. However, the current criterion is based on logic expression, instead of dynamic expression, which does not work well on high latitude location. The arc of vision versus azimuth criterion assessment founds that it primarily has high success rates in predicting positive lunar crescent sighting for Fotheringham and Maunder criterion and works well for prediction of negative lunar crescent sighting for Fatoohi and Ilyas. The azimuth-arc of vision criterion found diminishing popularity in recent years, therefore it does not include critical optical aided observation. Bruin has pioneered the usage of width in lunar crescent visibility criterion, and while Yallop sparked the interest of multirange lunar crescent visibility criterion among researchers, such as Odeh, and

Qureshi. The width lunar crescent visibility criterion is inconsistent from one lunar cycle to another, making it a less reliable criterion in predicting lunar crescent visibility. Lag time lunar crescent visibility criterion found its criticism among researchers in the late 20th century, however Gautschy able to demonstrate the strength of lag time variable in predicting lunar crescent sighting, particularly for naked eye observation. This paper also found that there are at least 2 % errors in locating visibility of lunar crescent among expert observers, echoing the finding of Doggett et al. in 1994. The paper also provides an alternative lunar crescent visibility criterion for each of the criterion models, this hope to ignite an engagement on determining the best model of lunar crescent sighting.

CRediT authorship contribution statement

Muhamad Syazwan Faid: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Project administration, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Mohd Saiful Anwar Mohd Nawawi:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Mohd Hafiz Mohd Saadon:** Project administration, Methodology, Formal analysis, Data curation. **Raihana Abdul Wahab:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision. **Nazhatulshima Ahmad:** Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Muhamad Syauqi Nahwandi:** Supervision, Software, Resources, Project administration, Methodology. **Ikramullah Ahmed:** Writing – review & editing, Writing – original draft, Visualization, Formal analysis, Data curation, Conceptualization. **Ibrahim Mohamed:** Visualization, Software, Data curation.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Mohd Saiful Anwar Mohd Nawawi reports financial support was provided by University of Malaya. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgements

This research is supported by the University of Malaya Grant, IIRG002D-19FNW, RMF0068-2021,UMG004L-2022 and IIRG002A-19FNW. We would like to express our sincere gratitude to Patrick Rocher, former astronomer at the observatory of Paris and official advisor of the Paris Mosque for astronomical issues. His invaluable expertise and thoughtful insights as a reviewer significantly contributed to the refinement and enhancement of this manuscript. We appreciate his dedicated commitment to advancing the quality and accuracy of the content. However, we acknowledge that the decision to include this acknowledgment is at the discretion of the authors.

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